

Parts of the Cubit, and related lengths

Parts of the cubit, using π , ϕ , e , Royal Cubit \mathcal{C} , Grand Metre \mathcal{M} , and roots/squares/cubes
All based on a circle with diameter 1 metre

In formulas below, see π as being a length of 3.14159... metres, not just the number pi.
Similarly with ϕ and e .

Unless otherwise specified, all subdivisions are in “Egyptian” units, so one Foot is one Egyptian Foot, etc.

$$\mathcal{M} = 1 + \mathcal{C} = 1 + \frac{\pi}{6} m$$

The tables are arranged by “method” to more clearly show the patterns involved in getting the different lengths, using basic arithmetic and well-known irrational constants.

I took the measurements from [Wikipedia's page on ancient Egyptian measurements](#) but think their measurement for the royal cubit is wrong. Their measurements are similar to this, which is from [The cadastral survey of Egypt 1892-1907](#) which only goes to three decimal places, and again, I don't like their royal cubit measurement.

7 : 6 : 5. As subdivisions of the cubit there were the “span,” the “handsbreadth,” and the “finger,” this last being even subdivided as low as one-sixteenth. These subdivisions are derived from several inscribed examples of cubit rods which have been found in ancient tombs and a series of them has been described by Lepsius in a memoir on the subject.† He gives the following fractions of a cubit ‡ :—

		palms.	metre.
meh suten	or royal cubit	= 7	= 0.525
meh net's	„ short cubit	= 6	= 0.450
remen	„ upper arm	= 5	= 0.375
t' eser		= 4	= 0.300
pet aa	„ great span	= $3\frac{1}{2}$	= 0.262
pet net's	„ small span	= 3	= 0.225
	handsbreadth	= $1\frac{1}{4}$	= 0.094
shep	„ palm	= 1	= 0.075
t'ba	„ digit	= $\frac{1}{4}$	= 0.019

* SCHÄFER. Abhandlungen d. königl. preuss. Akademie 1902 ;

NAVILLE. “Recueil de travaux archéologiques.” 1903, p. 64.

† “Ueber die altägyptische Elle und ihre Eintheilung.” Berlin, 1865.


‡ See also GRIFFITH, “Proc. Soc. Bib. Archæology theilung,” 1892, p. 433 ff. for much information on Egyptian weights and measures.

These measurements are literally measurement based on various cubit rods, which are subject to copying errors over time as well as thickness of pen, brush or chisel used. There is also an alternative length for the remen, given variously as [0.3701](#) to .3703, I think the real length is 0.3702. The problem with measuring things is dealing with the 4th decimal place (0.0001m which is 1/10th of a millimeter)... a distance which is less than the diameter of a grain of sand.

Based on \mathcal{M}

Length	Value	Formula	Value
Digit	0.01875 m	$\frac{\mathcal{M}}{16 \pi \varphi}$	0.01873 m
Palm	0.0750 m	$\frac{\mathcal{M}}{4 \pi \varphi}$	0.0749 m
		$\frac{\mathcal{M}}{9 \sqrt{\pi \varphi}}$	0.0751 m
Hand	0.0938 m	$\sqrt{\frac{\mathcal{M}}{100 \sqrt{3}}}$	0.0938 m
Fist	0.1125 m	$\frac{\mathcal{M}}{6 \sqrt{\pi \varphi}}$	0.1126 m
Double Handbreadth	0.1500 m	$\frac{\mathcal{M}}{2 \pi \varphi}$	0.1499 m
		$\frac{2 \mathcal{M}}{9 \sqrt{\pi \varphi}}$	0.1502 m
Small span	0.2250 m	$\frac{\mathcal{M}}{3 \sqrt{\pi \varphi}}$	0.2253 m
Great span	0.2618 m	$\frac{\mathcal{M} \varphi}{3 \pi}$	0.2616 m
Foot	0.3000 m	$\frac{\mathcal{M}}{\pi \varphi}$	0.2997 m
		$\frac{4 \mathcal{M}}{9 \sqrt{\pi \varphi}}$	0.3003 m
Remen	0.3703 m		
Remen	0.3750 m	$\frac{5 \mathcal{M}}{4 \pi \varphi}$	0.3747 m
Cubit (standard)	0.4500 m	$\frac{3 \mathcal{M}}{2 \pi \varphi}$	0.4496 m
		$\frac{2 \mathcal{M}}{3 \sqrt{\pi \varphi}}$	0.4505 m
Cubit (royal) \mathcal{G}	0.5236 m	$\frac{2 \mathcal{M} \varphi}{3 \pi}$	0.5231 m
Pole	0.6000 m	$\frac{2 \mathcal{M}}{\pi \varphi}$	0.5995 m
		$\frac{\mathcal{M}}{2 \sqrt{\varphi}}$	0.5989 m
Nby-Rod	0.67 – 0.68 m	$\frac{4 \mathcal{M}}{9}$	0.6772 m


Based on $\frac{\pi}{\varphi^2}$

Length	Value	Formula	Value
Digit	0.01875 m	$\frac{\pi}{64 \varphi^2}$	0.01875 m
Palm	0.0750 m	$\frac{\pi}{16 \varphi^2}$	0.0750 m
Hand	0.0938 m	$\frac{5 \pi}{64 \varphi^2}$	0.0938 m
Fist	0.1125 m	$\frac{9 \pi}{96 \varphi^2}$	0.1125 m
Double Handbreadth	0.1500 m	$\frac{\pi}{8 \varphi^2}$	0.1500 m
Small span	0.2250 m	$\frac{3 \pi}{16 \varphi^2}$	0.2250 m
Great span	0.2618 m	$\frac{\pi - \varphi^2}{2}$	0.2618 m
Foot	0.3000 m	$\frac{\pi}{4 \varphi^2}$	0.3000 m
Remen	0.3703 m		
Remen	0.3750 m	$\frac{5 \pi}{16 \varphi^2}$	0.3750 m
Cubit (standard)	0.4500 m	$\frac{3 \pi}{8 \varphi^2}$	0.4500 m
Cubit (royal) 	0.5236 m	$\pi - \varphi^2$	0.5236 m
Pole	0.6000 m	$\frac{\pi}{2 \varphi^2}$	0.6000 m
Nby-Rod	0.67 – 0.68 m	$\frac{9 \pi}{16 \varphi^2}$	0.6750 m


Based on $\frac{\varphi e}{\pi}$ or $\frac{\varphi \pi}{e}$

Length	Value	Formula	Value
Digit	0.01875 m	$\frac{3 \varphi e}{224 \pi}$	0.01875 m
Palm	0.0750 m	$\frac{3 \varphi e}{56 \pi}$	0.0750 m
Hand	0.0938 m	$\frac{15 \varphi e}{224 \pi}$	0.0938 m
Fist	0.1125 m	$\frac{18 \varphi e}{224 \pi}$	0.1125 m
Double Handbreadth	0.1500 m	$\frac{3 \varphi e}{28 \pi}$	0.1500 m
Small span	0.2250 m	$\frac{9 \varphi e}{56 \pi}$	0.2250 m
Great span	0.2618 m	$\frac{7 \varphi \pi}{50 e}$	0.2618 m
Foot	0.3000 m	$\frac{3 \varphi e}{14 \pi}$	0.3000 m
Remen	0.3703 m		
Remen	0.3750 m	$\frac{15 \varphi e}{56 \pi} = \frac{(7+8) \varphi e}{(7 \times 8) \pi}$	0.3750 m
Cubit (standard)	0.4500 m	$\frac{9 \varphi e}{28 \pi}$	0.4500 m
Cubit (royal) 	0.5236 m	$\frac{7 \varphi \pi}{25 e}$	0.5236 m
Pole	0.6000 m	$\frac{3 \varphi e}{7 \pi}$	0.6000 m
Nby-Rod	0.67 – 0.68 m	$\frac{27 \varphi e}{56 \pi}$	0.6750 m
		$36 \frac{3 \varphi e}{224 \pi}$ (e.g.)	0.6750 m (36 digits)


Based on $\left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1\right)$

Length	Value	Formula	Value
Digit	0.01875 m	$\frac{1}{16} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.01875 m
Palm	0.0750 m	$\frac{1}{4} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.0750 m
Hand	0.0938 m	$\frac{5}{16} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.09375 m
Fist	0.1125 m	$\frac{3}{8} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.1125 m
Double Handbreadth	0.1500 m	$\frac{1}{2} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.1500 m
Small span	0.2250 m	$\frac{3}{4} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.2250 m
Great span	0.2618 m	$\frac{1}{3} \varphi^2 \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.2618 m
Foot	0.3000 m	$\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1$	0.3000 m
Remen	0.3703 m		
Remen	0.3750 m	$\frac{5}{4} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.3750 m
Cubit (standard)	0.4500 m	$\frac{3}{2} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.4500 m
Cubit (royal) 	0.5236 m	$\frac{2}{3} \varphi^2 \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.5236 m
Pole	0.6000 m	$2 \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.6000 m
Nby-Rod	0.67 – 0.68 m	$\frac{9}{4} \left(\frac{\sqrt{\pi^2 + \varphi^2}}{e} - 1 \right)$	0.3750 m


Based on $\frac{\sqrt{2}}{\pi}$

Length	Value	Formula	Value
Digit	0.01875 m	$\frac{\sqrt{2}}{24 \pi}$	0.01876 m
Palm	0.0750 m	$\frac{\sqrt{2}}{6 \pi}$	0.0750 m
Hand	0.0938 m	$\frac{5\sqrt{2}}{24 \pi}$	0.0938 m
Fist	0.1125 m	$\frac{\sqrt{2}}{4 \pi}$	0.1125 m
Double Handbreadth	0.1500 m	$\frac{\sqrt{2}}{3 \pi}$	0.1501 m
Small span	0.2250 m	$\frac{\sqrt{2}}{2 \pi}$	0.2251 m
Great span	0.2618 m	$\frac{2\sqrt{2} \varphi^2}{9 \pi}$	0.2619 m
Foot	0.3000 m	$\frac{2\sqrt{2}}{3 \pi}$	0.3001 m
Remen	0.3703 m		
Remen	0.3750 m	$\frac{5\sqrt{2}}{6 \pi}$	0.3751 m
Cubit (standard)	0.4500 m	$\frac{\sqrt{2}}{\pi}$	0.4502 m
Cubit (royal) 	0.5236 m	$\frac{4\sqrt{2} \varphi^2}{9 \pi}$	0.5238 m
Pole	0.6000 m	$\frac{4\sqrt{2}}{3 \pi}$	0.6002 m
Nby-Rod	0.67 – 0.68 m	$\frac{3\sqrt{2}}{2 \pi}$	0.6752 m

Based on π

Digits	Length	Value	Formula	Value
1	Digit	0.01875 m	$\frac{\pi}{7 \times 24} = \frac{\pi}{168}$	0.0187 m
4	Palm	0.0750 m	$\frac{\pi}{7 \times 6} = \frac{\pi}{42}$	0.0748 m
5	Hand	0.0938 m	$\frac{5\pi}{7 \times 24} = \frac{5\pi}{168}$	0.0935 m
6	Fist	0.1125 m	$\frac{\pi}{7 \times 4} = \frac{\pi}{28}$	0.1122 m
8	Double Handbreadth	0.1500 m	$\frac{\pi}{7 \times 3} = \frac{\pi}{21}$	0.1496 m
12	Small span	0.2250 m	$\frac{\pi}{7 \times 2} = \frac{\pi}{14}$	0.2244 m
14	Great span	0.2618 m	$\frac{\pi}{6 \times 2} = \frac{\pi}{12}$	0.2618 m
16	Foot	0.3000 m	$\frac{2\pi}{7 \times 3} = \frac{2\pi}{21}$	0.2992 m
	Remen	0.3703 m	$\frac{\pi}{6\sqrt{2}}$	0.3702 m
20	Remen	0.3750 m	$\frac{5\pi}{7 \times 6} = \frac{5\pi}{42}$	0.3740 m
24	Cubit (standard)	0.4500 m	$\frac{\pi}{7}$	0.4488 m
28	Cubit (royal) 	0.5236 m	$\frac{\pi}{6}$	0.5236 m
32	Pole	0.6000 m	$\frac{4\pi}{7 \times 3} = \frac{4\pi}{21}$	0.5984 m
36	Nby-Rod	0.67 – 0.68 m	$\frac{3\pi}{7 \times 2} = \frac{3\pi}{14}$	0.6732 m

Based on $\frac{\sqrt{2}}{\pi}$

Length	Value	Formula	Value
Digit	0.01875 m	$\frac{\pi}{120\sqrt{2}}$	0.0185 m
Palm	0.0750 m	$\frac{\pi}{30\sqrt{2}}$	0.7405 m
Hand	0.0938 m	$\frac{\pi}{24\sqrt{2}}$	0.0926 m
Fist	0.1125 m	$\frac{\pi}{20\sqrt{2}}$	0.1111 m
Double Handbreadth	0.1500 m	$\frac{\pi}{15\sqrt{2}}$	0.1481 m
Small span	0.2250 m	$\frac{\pi}{10\sqrt{2}}$	0.2221 m
Great span	0.2618 m	$\frac{\pi}{2\sqrt{18}\sqrt{2}} = \frac{\pi}{2\sqrt{36}} = \frac{\pi}{12}$	0.2618 m
Foot	0.3000 m	$\frac{2\pi}{15\sqrt{2}}$	0.2962 m
Remen	0.3703 m	$\frac{\pi}{6\sqrt{2}}$	0.3702 m
Remen	0.3750 m		0. m
Cubit (standard)	0.4500 m	$\frac{\pi}{5\sqrt{2}}$	0.4443 m
Cubit (royal) 	0.5236 m	$\frac{\pi}{\sqrt{18}\sqrt{2}} = \frac{\pi}{\sqrt{36}} = \frac{\pi}{6}$	0.5236 m
Pole	0.6000 m	$\frac{4\pi}{15\sqrt{2}}$	0.5924 m
Nby-Rod	0.67 – 0.68 m	$\frac{3\pi}{10\sqrt{2}}$	0.6664 m

Assorted

Length	Value	Formula	Value
Great span	0.2618 m		= half Royal Cubit
		$\frac{\mathcal{C}}{2}$	0.2618 m
		$\frac{\pi}{12}$	0.2618 m
		$\frac{\sqrt{5}}{\pi e}$	0.2618 m
Foot	0.3000 m	$\frac{\sqrt[3]{3}\sqrt[3]{5}}{6\mathcal{C}\varphi^2}$	0.2999 m
		$\frac{e}{12\mathcal{C}\sqrt[3]{3}}$	0.3000 m
Remen	0.3703	$\frac{\mathcal{C}}{\sqrt{2}}$	0.3702 m
		$\frac{\pi}{6\sqrt{2}}$	0.3702 m
		$\frac{7\varphi\pi}{25e\sqrt{2}}$	0.3702 m
Cubit (royal) \mathcal{C}	0.5236 m	$\frac{\pi}{6}$	0.5236 m
		$\frac{1}{5}\left(\frac{7\pi}{5e}\right)^2$	0.5236 m
		$\frac{\varphi e}{8.4}$	0.5236 m
		$\left(\frac{1+\pi}{e}-1\right)$	0.5236 m
		$\frac{\varphi^2}{5}$	0.5236 m
Pole	0.6000 m		= 2 Foot
		$\frac{2\sqrt[3]{3}\sqrt[3]{5}}{6\mathcal{C}\varphi^2}$	0.5997 m
		$\frac{e}{6\mathcal{C}\sqrt[3]{3}}$	0.5999 m
Nby-Rod	0.67 – 0.68 m	Foot x $\sqrt{\pi\varphi}$	0.6764 m
		Cubit x $\frac{2\sqrt{\pi\varphi}}{3}$	0.6764 m
Grand metre \mathcal{M}	1.5326 m	$1+\frac{\pi}{6}$	1.5236 m
		$\frac{(1+\pi)}{e}$	1.5236 m

Length	Value	Formula	Value
1 metre	1.0000 m	$\frac{5\varphi e}{7\pi}$	1.000 m
4 Feet	1.2000 m	$\frac{\pi}{\varphi^2}$	1.2000 m
?	1.3000 m	$\frac{\sqrt{\pi^2 + \varphi^2}}{e}$	1.3000 m
7 Wroft	1.4000 m	$\frac{\varphi e}{\pi}$	1.4000 m
English foot	0.3048 m	$\frac{\mathcal{M}}{5}$	03047 m
		$\frac{1.524}{5}$	0.3048 m
Five English feet	60" = 1.5240 m	$1 + \frac{\pi}{6} = \mathcal{M}$	1.5236 m
Persian foot	0.32004 m	$\frac{\mathcal{M}}{(\pi + \varphi)}$	0.32011 m
Doric order foot	±0.324 m	$\frac{\pi}{6\varphi} = \frac{\mathcal{E}}{\varphi}$	0.3236 m
		$\frac{\pi}{\sqrt{2}\varphi^4}$	0.3241 m
Luwian foot	±0.323 m	$\frac{\pi}{6\phi} = \frac{\mathcal{E}}{\phi}$	0.3236 m
Attic foot	0.3084 m	$\sqrt{\frac{\mathcal{M}}{16}}$?	0.3086m ?
Minoan foot	+0.304 m	$\frac{\mathcal{M}}{5}$	0.3047 m
Athenian foot	±0.315 m	$\frac{\pi}{10}$	0.3142 m
Phoenician foot	0.3000 m	$\frac{\pi}{4\varphi^2} = \frac{3\varphi e}{14\pi}$	0.3000 m
Megalithic yard	0.8275 m	remen x root(5)	
Nautical mile	1852 m (currently)	$100\pi\varphi\left(\frac{1}{\mathcal{E}}\right)^2$	1854.1 m
		$100\pi\varphi\left(\frac{1}{0.524}\right)^2$	1851.3 m
		$\frac{3600\varphi}{\pi}$	1854.1 m
		$\frac{5040}{e} = \frac{7!}{e}$	1854.1 m